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Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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Introduction

Objective: quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

Approach: introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

Material and Methods

Affine transformation: \( y = Lx + t \)

\[ L = RSU^T \]

- \( U \) is a rotation matrix that determines scaling directions
- \( S \) is an anisotropic scaling (diagonal matrix)
- \( R \) is a rotation matrix

Anisotropic similarity: affine transformation with fixed \( U \)

Block-matching registration [1][2]:

1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:

Minimize: \( C(R, S, t) = \sum_i \| y_i - (RSU^T x_i + t) \|^2 \), \( U \) fixed

\[ \bar{t} = \bar{g} - RSU^T \bar{x} \]

\[ \tilde{C}(R, S) = \sum_i \| y_i' - RS\bar{x}_i \|^2 \]

\( y_i' = y_i - \bar{g}, \bar{x} = U^T(x_i - \bar{x}) \)

\[ \tilde{C}(q, S) = \sum_i \| y_i' - q \xi_i \|^2 \]

\( \xi_i = S\bar{x}_i \) [3]

Matricial quaternions: \( y_i' q = Q_{y_i} q \) and \( -q \xi_i = -P_{\xi_i} q = P_{\xi_i} q \)

\[ \tilde{C}(q, S) = -q^T \left( -\sum_i (Q_{y_i} + P_{\xi_i})^2 \right) q \]

\( A_i = (Q_{y_i} + P_{\xi_i})^2 \) and \( A = \sum_i A_i \)

Alternate optimization each having an analytical solution:

- For fixed \( S \), \( \tilde{q} \) eigenvector with smallest eigenvalue of \( A \)
- For fixed \( q \), \( \tilde{s}_{jj} = 1 \sum_i \bar{x}_j q^T \left( \sum_i Q_{y_i} \frac{\partial P_{\xi_i}}{\partial s_j} \right) q \)

We registered whole brain and lobes from 308 subjects onto an atlas based on this population using anisotropic similarity with chosen \( U \).

Then we extracted the associated scaling factors describing brain growth.

Results

Scaling factors along chosen directions

Black curve represents average model

Fitted using first order rational regression: \( y = \frac{ax + b}{x + c} \)

Scaling factors over age in years

whole brain

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.

An interesting continuation is to compare the results among groups of population.

References