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Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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Introduction

**Objective:** quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

**Approach:** introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

Material and Methods

Affine transformation: \( y = Lx + t \)
\( L = RSU^T \)
- \( U \) is a rotation matrix that determines scaling directions
- \( S \) is an anisotropic scaling (diagonal matrix)
- \( R \) is a rotation matrix

Anisotropic similarity: affine transformation with fixed \( U \)

Block-matching registration [1][2]:
1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:
Minimize: \( C(R, S, t) = \sum_i \| y_i - (RSU^T x_i + t) \|^2 \), \( U \) fixed
\[ \hat{t} = \hat{g} - RSU^T \hat{x} \]
\[ \hat{C}(R, S) = \sum_i \| y_i' - RS\hat{x}_i \|^2 \]
\[ y_i' = y_i - \hat{g}, \hat{x} = U^T (x_i - \hat{x}) \]
\[ \hat{C}(q, S) = \sum_i \| y_i' - q \circ \xi_i \|^2 \]
\[ \xi_i = S\hat{x}_i \] [3]

Matricial quaternions: \( y_i' \circ q = Q_{y_i}q \) and \(-q \circ \xi_i = -P_{\xi_i}q = P_{\xi_i}q\)
\[ \hat{C}(q, S) = -q^T \left( -\sum_i (Q_{y_i} + P_{\xi_i})^2 \right) q \]
\[ A_i = (Q_{y_i} + P_{\xi_i})^2 \] and \( A = \sum_i A_i \).

Alternate optimization each having an analytical solution:
- For fixed \( S \), \( \hat{q} \) eigenvector with smallest eigenvalue of \( A \)
- For fixed \( q \), \( \tilde{S}_{ij} = \frac{1}{\sigma_i^2} q^T \sum_i Q_{y_i} \frac{\partial P_{\xi_i}}{\partial s_{ij}} q \)

Results

Scaling factors along chosen directions
Black curve represents average model
Fitted using first order rational regression: \( y = \frac{ax + b}{x + c} \)

Chosen scaling directions fixed on the reference image:
1. Orthogonal to the mid-sagittal plane (blue) [4]
2. and 3. Principal directions of voxel coordinates projected on mid-sagittal plane (red and green)

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.

An interesting continuation is to compare the results among groups of population.