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Anisotropic similarity, a constrained affine transformation: Application to brain development analysis

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Introduction

Objective: quantify regional brain growth in 3 orthogonal directions by linearly registering subjects on a common basis.

Approach: introduce an affine transformation with constrained scaling directions (9 degrees of freedom).

Material and Methods

Affine transformation: \( y = Lx + t \)
\( L = RSU^T \)
- \( U \) is a rotation matrix that determines scaling directions
- \( S \) is an anisotropic scaling (diagonal matrix)
- \( R \) is a rotation matrix

Anisotropic similarity: affine transformation with fixed \( U \)

Block-matching registration [1][2]:
1. Matching: find homologous blocks in both images best satisfying a similarity criterion
2. Aggregation into a global transformation by minimizing the distance between the sets of blocks

For an anisotropic similarity:
Minimize: \( C(R, S, t) = \sum |y_i - (RSU^T x_i + t)|^2 \), \( U \) fixed
\[
\hat{t} = \bar{y} - R\bar{S}U^T\bar{x}
\]
\[
\hat{C}(R, S) = \sum |y'_i - RS\hat{x}|^2
\]
\[
\hat{C}(q, S) = \sum |y'_i - q * \xi_i| \quad \xi_i = S\hat{x}_i \quad [3]
\]

Matricial quaternions: \( y'_i \ast q = Q_{y'_i}q \) and \(-q * \xi_i = -P_{\xi_i}q = P_{\xi_i}q \)
\[
\hat{C}(q, S) = -q^T \left( -\sum_i (Q_{y'_i} + P_{\xi_i})^2 \right) q
\]
\( A_i = (Q_{y'_i} + P_{\xi_i})^2 \) and \( A = \sum_i A_i \).

Alternate optimization each having an analytical solution:
- For fixed \( S \), \( \frac{q}{\bar{q}} \) eigenvector with smallest eigenvalue of \( A \)
- For fixed \( q \), \( \hat{S}_{ij} = \frac{1}{\sum_i \hat{x}_i^2} q^T \left( \sum_i Q_{y'_i} \frac{\partial P_{\xi_i}}{\partial s_j} \right) q \)

We registered whole brain and lobes from 308 subjects onto an atlas based on this population using anisotropic similarity with chosen \( U \).

Then we extracted the associated scaling factors describing brain growth.

Results

Scaling factors along chosen directions
Black curve represents average model
Fitted using first order rational regression: \( y = \frac{ax + b}{x + c} \)

Chosen scaling directions fixed on the reference image:
1. Orthogonal to the mid-sagittal plane (blue) [4]
2. and 3. Principal directions of voxel coordinates projected on mid-sagittal plane (red and green)

Conclusion

We developed a method to quantify regional brain growth in 3 orthogonal directions through anisotropic similarity registration.
An interesting continuation is to compare the results among groups of population.