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Abstract — The well-known calculation of Womersley for the pulse wave propagation in a deformable artery is revisited here, through the inclusion of the magnetic force in the Navier-Stokes equations.

Index terms - Biomechanics, Medical physics, Modeling

I. INTRODUCTION

Several biomedical applications require a good knowledge of the physics of blood flow and pressure pulse propagation in a deformable vessel, in the presence of an external static magnetic field:

- In the case of Magnetic Resonance Imaging (MRI) of the heart, the charged particles of the blood get deflected by the Lorentz force, thus inducing electrical currents and voltages across the vessel wall and in the surrounding tissues. These voltages disturb the electrocardiogram (ECG) detected at the surface of the thorax, making the ECG-based image synchronization inaccurate. Several approaches have been proposed to correct the ECG signals for these magnetohydrodynamic (MHD) induced artifacts [1]. On the contrary, other groups [2], decided to try to use the MHD effect measured with ECG as a biomarker of cardiac output, since it is sensitive to magnetic flux density, flow orientation with respect to the magnetic field lines, and velocity of the blood. Frauenrath et al. [3] even propose to use the MHD induced voltage itself as a tool for cardiac synchronization.
- Pulse wave velocity (PWV) is known to be an indicator of arterial stiffness, and, as a matter of fact, a predictor of cardiovascular risk. MRI has been proposed by several groups as a noninvasive tool to assess PWV [4].
- Quite recently, a new direction of research has emerged, aiming at harvesting intracorporeal energy to provide continuous power to implanted medical devices. The advantage would be to avoid costly and tedious replacement or recharging of these medical implants. Pfenniger et al. [5] propose a prototype that converts the expansion and contraction of an artery (due to pulsatile blood pressure changes) into electrical energy by electromagnetic induction.

For a Newtonian incompressible fluid, the MHD equations are defined by a coupling of Maxwell's electromagnetic equations and Ohm's law, on the one hand, and the Navier-Stokes equations including the Lorentz force on the other hand. An optimal modelisation of the magnetohydrodynamic flow of blood should include the pulsatility of flow, the deformability and conductivity of the vessel wall, together with the induced electrostatic and electromagnetic fields. This leads to a complex mathematical problem and analytical solutions may be found only under restrictive hypotheses. To our knowledge, the solutions that may be found in the literature assume that the vessel is rigid. The present work provides a solution including the vessel wall deformability by a coupling of fluid equations and equations for the motion of the wall, in the case of simple sinusoidal flow, non-conducting wall and neglected inductions.

II. THEORY

The calculations are conducted following the method proposed by Womersley [6] and by Atabek and Lew [7]. The main velocity component of the blood is longitudinal (along z). The radial component (which is related to the wall deformability) is small, when compared with the longitudinal component. The vessel wall is assumed to be thin, elastic and isotropic. The analysis is restricted to waves whose wavelengths are very large compared with the radius of the tube, and it is further assumed that the amplitude of the pressure disturbance is sufficiently small so that non-linear terms of the inertia of the fluid are negligible compared with linear ones. The $\mathbf{B_0}$ field is transverse to the vessel. The Lorentz force acting on the fluid is expressed as $\mathbf{j}$^$\mathbf{B_0}$, where $\mathbf{j}$ is the electric current density, given by $\sigma(\mathbf{u}$^$\mathbf{B_0})$ (σ is the electric conductivity of the blood, and $\mathbf{u}$ is the fluid velocity field). The equations describing the motion of the wall, and the boundary conditions between the fluid and the deformable wall are the same as in [7]. The solutions of this system of equations are searched for the case of the propagation of forced pressure waves which are harmonic in t (time) and in z. All the variables (pressure, radial and longitudinal fluid velocities and wall displacements) take the form

$X(r,t,z)=X^*(r)\exp[i\omega(t-(z/c))]$,

with i the complex number such that $i^2 = -1$, ω the circular frequency of the forced oscillations, and c, the velocity (complex) of propagation of the oscillations. A frequency equation, which is quadratic in the square of the propagation velocity, is obtained. Two out of four roots of this equation give the velocity of propagation of two distinct outgoing waves. The remaining two roots represent incoming waves corresponding to the first two waves. One of the waves propagates more slowly ("Young mode") than the other ("Lamb mode"). This last type of waves is essentially longitudinal waves through the tube wall, modified with the existence of the fluid.

III. RESULTS

The influence of the magnetic field intensity and of the amplitude of the pressure perturbation on the two fluid velocity components, on the two wall displacement components and on the wave celerity is demonstrated (see for example Figure1). The magnetic field induces some flow reduction and some flattening of the fluid velocity profiles. It also induces a reduction of the wave celerity of the Young mode. These effects heighten when B_0 increases. However, in the range of the values of B_0 used in routine MRI (B_0 <10 Tesla), they remain quite negligible.

The influence of the wall deformability can also be established through the comparison of the present solution with the solution of Sud et al. [8], obtained with the same hypotheses except that the tube is rigid.

IV. DISCUSSION – CONCLUSION

In spite of the numerous simplifying hypotheses, it seems that this solution allows to capture some important features of the physical problem. It is also hoped that such analytical solutions may be used to validate the 3D computational fluid dynamics codes that may be developed by other groups.

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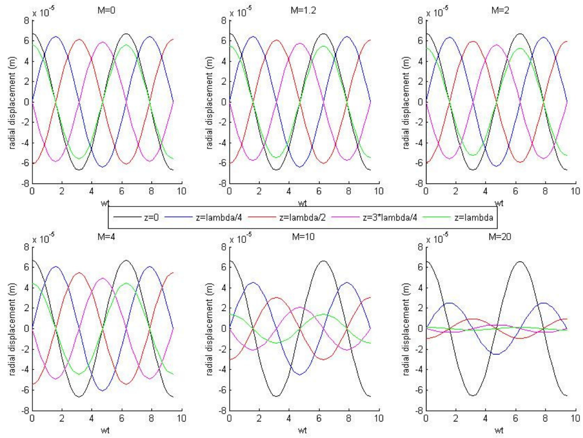

Figure 1: Propagation and damping of the radial displacement of the wall, for the « Young mode » wave celerities, obtained for a 10 mmHg pressure perturbation in the vessel. The displacements (full scale = 80 microns) are plotted against (ωt) (in radians), for different longitudinal positions ($z = 0$; $z = \lambda/4$; $z = \lambda/2$; $z = 3\lambda/4$; $z = \lambda$). The influence of the magnetic field, B_0, is included in the non-dimensional Hartmann number, M, defined as: $M = RB_0\,((\sigma/\eta)^{1/2})$, where R is the "mean" radius of the vessel (1 cm), σ the blood electric conductivity (0.5 S/m), and η its dynamic viscosity (4 mPa.s). M= 1.2 thus corresponds to B_0 = 10.7 Teslas, M = 10 corresponds to B_0 = 89.4 T,,... The other data used here were: blood density = 1050 kg/m^3, wall density = 1100 kg/m^3, frequency = 75 b.p.m., Young modulus of the vessel wall = 10^6Pa, wall thickness = 2 mm, Poisson coefficient = 0.5.