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Assessing the limitations of the Banister model in monitoring training

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Assessing the limitations of the Banister model in monitoring training

Running title: Variability in the Banister model.

Key words: bootstrap method, inaccuracy, instability, training loads, swimming.

Abstract

The aim of this study was to carry out a statistical analysis of the Banister model to verify how useful it is in monitoring the training programmes of elite swimmers. The accuracy, the ill-conditioning and the stability of this model were thus investigated. Training loads of nine elite swimmers, measured over one season, were related to performances with the Banister model. Firstly, to assess accuracy, the 95% bootstrap confidence interval (95% CI) of parameter estimates and modelled performances were calculated. Secondly, to study ill-conditioning, the correlation matrix of parameter estimates was computed. Finally, to analyse stability, iterative computation was performed with the same data but minus one performance, chosen randomly. Performances were significantly related to training loads in all subjects ($R^2 = 0.79 \pm 0.13$, $P < 0.05$) and the estimation procedure seemed to be stable. Nevertheless, the 95% CI of the most useful parameters for monitoring training were wide $\tau_a = 38$ (17, 59), $\tau_f = 19$ (6, 32), $t_n = 19$ (7, 35), $t_g = 43$ (25, 61). Furthermore, some parameters were highly correlated making their interpretation worthless. The study suggested possible ways to deal with these problems and reviewed alternative methods to model the training-performance relationships.

Introduction

The relationships between training and performance may be of practical use to sports elite coaches when organizing their athletes' training programs. These relationships are known to be highly individualized (Mujika *et al.*, 1996b; Avalos *et al.*, 2003). These differences can be attributed to genetic factors (Wolfarth *et al.*, 2000), individual training background (Mujika *et al.*, 1996a; Avalos *et al.*, 2003), psychological factors (Banister *et al.*, 1975), technical factors (Toussaint *et al.*, 1994; Wakayoshi *et al.*, 1995) and specialty (Mujika *et al.*, 1996a,b; Stewart and Hopkins, 2000; Avalos *et al.*, 2003). There is a large consensus that modelling training-performance relations provides pertinent information concerning inter-individual differences enabling highly individualised training programmes (Banister *et al.*, 1975, 1999; Morton *et al.*, 1990, 1991, 1997; Mujika *et al.*, 1996a, b; Avalos *et al.*, 2003). The model proposed by Banister *et al.* (1975) and its different extensions (Calvert *et al.*, 1976; Busso *et al.*, 1997, 2002; Busso, 2003) aimed to relate training loads to performance, taking into account the dynamic and temporal characteristics of training and therefore the effects of load sequences over time. These effects may be described by two antagonistic transfer functions: first, a positive influence that synthesizes all the positive effects leading to an increase in performance, second a negative function that synthesizes all the negative effects leading to short- or long-term fatigue and having a negative influence on performance (Busso *et al.*, 1994; Mujika *et al.*, 1996a). The function is as follows (Banister *et al.*, 1975; Busso *et al.*, 1994):

$$p_t = p_0 + k_a \sum_{s=0}^{t-1} e^{-(t-s)/\tau_a} w_s - k_f \sum_{s=0}^{t-1} e^{-(t-s)/\tau_f} w_s$$

where p_t is the modelled performance at time t ; p_0 is the initial performance level; k_a and k_f are the fitness and fatigue magnitude factor, respectively; τ_a and τ_f are the fitness and fatigue decay time constant, respectively; and w_t is the known training load per week (or day) from the first week of training to the week (or day) preceding the performance. These parameters were interpreted as individual responses profiles (Mujika *et al.*, 1996a), presented as usable within the training prescription; *e.g.* t_n , the time to recover performance and t_g , the time to peak performance after training completion (Fitz-Clarke *et al.*, 1991). However, several authors (Busso *et al.*, 1990; Mujika *et al.*, 1996a; Taha and Thomas, 2003) reported that the practical interpretation of the positive and negative influences might be difficult. For example, Busso *et al.* (1990) reported a positive correlation between the testosterone level and the function of fatigue, while a negative relationship was expected. Recently, Taha and Thomas (2003) criticized the different models stemming from the original Banister model (Banister *et al.*, 1975), stressing (i) the inability of the model to accurately predict future performance; (ii) the differences between the estimated time course of change in performance and experimental observations; (iii) the fact that most of these models were poorly corroborated by the physiological mechanisms.

Moreover, quality measures of models were commonly limited to determination coefficient (R^2) associated with the F -ratio test. R^2 evaluates the goodness of fit but does not necessarily guarantee accurate prediction (Bartley, 1997; Atkinson and Nevill, 1998). With the F -ratio test, the hypothesis H_0 : $k_a = 0$ and $k_f = 0$ and $\tau_a = 0$ and $\tau_f = 0$ is tested versus H_1 : $k_a \neq 0$ and/or $k_f \neq 0$ and/or $\tau_a \neq 0$ and/or $\tau_f \neq 0$ (Sen and Shrivastava, 1990). A significant F -test implies that the Banister model is better than the constant model, $p_t = p_0$, in which training load does not affect performance.

Complementary measures are needed to indicate accuracy and sensitivity (Wetherill *et al.*, 1986; Efron and Tibshirani, 1993). Thus 95% CI's quantifies accuracy of parameter estimates and modelled performances. The asymptotic correlation matrix of parameter estimates assesses ill-conditioning and variability, which affect the accuracy and precision of parameter estimates (Bates and Watts, 1988; Belsley, 1991; Arsac *et al.*, 2004). A different criterion of quality concerns stability of the estimation procedure. A method is unstable if small perturbations in the data can cause significant changes in the estimations (Breiman, 1996).

Another important question is how many data points are needed per parameter to enable pertinent statistical analysis. For multiple linear regression, a nominal number of 15 observations per parameter (except the intercept parameter) is recommended (Stevens, 1986). But since the Banister model is non-linear, inference is based on asymptotic theory (Bates and Watts, 1988; Sen and Shrivastava, 1990; Davidian and Giltinan, 1995; Huet *et al.*, 1996), which implies more data points per parameter than for a linear regression model. In fact, the studies having modelled “real” performances in elite sport did not exceed more than 20 performances per year (Mujika *et al.*, 1996a; Millet *et al.*, 2002).

The problem of accuracy when modelling the training-performance relationship is crucial in high-level athletes. The higher the performance level, the smaller the difference in performance. For example, during the Athens Olympic Games, the mean difference in the swimming finals was $2.16 \pm 0.75\%$ between the Olympic Champion and the poorest (8th) performance, while for the national championship, this difference was $6.55 \pm 2.32\%$.

Therefore, the aims of the present study were 1) to assess the goodness of fit, accuracy, ill-conditioning and stability of the Banister model for real data; 2) To

review and suggest alternative methods to model the training-performance relationships.

Methods

Subjects

Nine (five females, four males) elite swimmers participated in the present study. All subjects were of international level (*e.g.* qualified as junior or elite in the national team for the European, World Championships or Olympic Games). Written informed consent was obtained from the subjects. Their training characteristics and performances were analyzed over a whole season composed of a training period (52 weeks) and a rest period (8 weeks) (see Table 1). Individualised training programmes were prescribed by coaches for each swimmer, depending on age, training background, individual profiles and specialty. The proportion of the training performed at each intensity level was compared between the group of sprint swimmers (SS) specialised in 50 and 100m events, the group of intermediary distance swimmers (IS) specialised in 100 and 200m events, and the group of middle distance swimmers (MDS) specialised in 200 and 400m events. The season was composed of four training cycles. A linear model of periodization characterized the training cycles (Fry *et al.*, 1992; ACSM, 2002): each training cycle, lasting between 8 and 14 weeks, commenced with high training volume and low intensity. As training progressed, volume decreased and intensity increased. The last three weeks prior the competitive period was defined as the taper phase.

**** Table 1 near here ****

Training and performance

An incremental stepwise test to exhaustion (6 x 200-m) was repeated four times in the season to determine the relationship between blood lactate concentration and swimming velocity. Then, intensity levels for swim workouts [below ($I1 \approx 2 \text{ mmol}\cdot\text{l}^{-1}$), equal to ($I2 \approx 4 \text{ mmol}\cdot\text{l}^{-1}$), slightly above ($I3 \approx 6 \text{ mmol}\cdot\text{l}^{-1}$) the onset of blood lactate accumulation, respectively; $I4 \approx 10 \text{ mmol}\cdot\text{l}^{-1}$; $I5 = \text{maximal intensity}$] were determined as proposed by Mujika *et al.* (1996a). $I6$ and $I7$ consisted in 40-70% and 70%-100% of 1 maximal repetition strength training, respectively.

For the whole group, the content of the volume, intensity and taper phases of the last training cycle before the main competitions of the year were compared (Table 2). Quantification of the training load was performed as indicated by Avalos *et al.* (2003); briefly, it was expressed as percentage of the maximal volume measured at each intensity level throughout the whole period studied for each subject.

For each swimmer, performances were measured during real competitions for the same event, during the entire study period. Since performance, in the case of swimming, is represented by a time, it appears simpler to operate in terms of the percentage of the best performance $\min_t(P_t)$ achieved in the course of the studied period. The performance can thus be computed in the following manner:

$$p_t = \frac{\min_t(P_t)}{P_t} * 100.$$

Performances achieved during the competition were compared with those achieved three weeks earlier during a preparatory competition.

Fitting the model

The model proposed by Banister *et al.* (1975) was used. Model parameters were estimated for each subject using the non-linear least squares iterative method, by

minimizing the residual sum of quadratic differences between the real and the modelled performances (RSS) with a Gauss-Newton type algorithm (Bates and Watts, 1988; Sen and Shrivastava, 1990; Davidian and Giltinan, 1995; Huet *et al.*, 1996). The values of parameters were generally reported as $\tau_a = 45$ days, $\tau_f = 15$ days, $k_a = 1$ arbitrary units (a.u.) and $k_f = 2$ a.u. (Morton *et al.*, 1990). However, in swimming, with similar swimmers and daily training loads, these values were $\tau_a = 41.4$ days, $\tau_f = 12.4$ days, $k_a = 0.128$ a.u. and $k_f = 0.055$ a.u. (Mujika *et al.*, 1996a). The latter were therefore retained as the initial values in the iterative procedure for each subject of the present study. Computations were completed using Matlab 2000 (version 6.0, Optimization Toolbox, Mathworks Eds). After testing the normality and homoscedasticity of the residuals, 95% CI were calculated for the estimated parameters. The determination coefficient was calculated as: $R^2 = 1 - (\text{RSS} / \text{TSS})$, where TSS is the total sum of squares. In addition, the statistical significance of the fit was tested by analysis of variance (*F*-ratio test). The average standard error values (ASE) were computed as the mean difference between modelled and real performances.

Calculation of t_n and t_g

The time to recover performance, t_n , and the time to peak performance after training

completion, t_g , were calculated as $t_n = \frac{\tau_1 \tau_2}{\tau_1 - \tau_2} \ln \frac{k_2}{k_1}$ and $t_g = \frac{\tau_1 \tau_2}{\tau_1 - \tau_2} \ln \left(\frac{k_2}{k_1} \frac{\tau_1}{\tau_2} \right)$

(subscripts “1” and “2” correspond to “a” and “f”) (Fitz-Clarke *et al.*, 1991).

Estimating the change in performance during the rest period

The time it took each swimmer to reach peak performance was evaluated during the rest period (during the final eight weeks of the season).

Accuracy

Bootstrap method was used to calculate the 95% CI of parameter estimates and modelled performances. Bootstrap is a powerful method for estimating tests statistics like CI, especially in small samples (Efron and Tibshirani, 1993; Huet *et al.*, 1996). Briefly, the procedure consisted of resampling the original data set with replacement to create 1000 “bootstrap replicate” data sets of the same size as the original data set. A random number generator was used to determine which data from the original data set to include in a replicate data set. Therefore a given data could be used more than once in the replicate data set, or not at all. This was repeated 1000 times. For each parameter, the estimates that fell between the 2.5th and the 97.5th percentiles of the 1000 estimates were used to construct a 95% CI for parameter estimates (τ_a , τ_f , k_a , k_f , t_n , t_g) and modelled performances. In addition the coefficient of variation (CV) was calculated as the ratio of the standard deviation over the mean of the 1000 replicates.

Ill-conditioning

Ill-conditioning is the nonlinear generalization of the collinearity problem in linear regression. The asymptotic correlation matrix of parameter estimates was checked to see whether any parameters were excessively highly correlated, since high correlations may reveal ill-conditioning problems, indicating that the model is over parameterised for the data set (Bates and Watts, 1988). The correlation matrix was obtained as follows. Let x_i be the explanatory variable and y_i the response variable, $i = 1, \dots, n$, where n is the number of observations. Suppose the relationship $y_i = f(x_i, \theta) + \varepsilon_i$, where the function f is the deterministic part of the model, depending on the explanatory variable and some parameters $\theta = (\theta_1, \dots, \theta_p)$, and ε_i is the random part of the model: $\varepsilon_i \sim N(0, \sigma^2)$. Let $\hat{\theta}$ be the non-linear least squares estimation, then, asymptotically, $\hat{\theta} \sim N(\theta, \hat{\sigma}^2 V)$, where $V = (D\hat{f}' D\hat{f})^{-1}$, $D\hat{f}$ is the derivative matrix

of \hat{f} with respect to θ , \hat{f} is the estimation of f , and $\hat{\sigma}^2$ is the standard unbiased estimator of the error variance. Elements of the correlation matrix, ρ_{jk} , $j, k = 1, \dots, p$, are then obtained from the covariance matrix as $\rho_{jk} = (V)_{jk} / [(V)_{jj}(V)_{kk}]^{1/2}$ (Bates and Watts, 1988; Sen and Shrivastava, 1990; Davidian and Giltinan, 1995; Huet *et al.*, 1996).

Stability

The stability of the Banister model fitted by minimizing the RSS value was studied as follows. For each subject, iterative computation was performed with the same data, but minus one performance, that was chosen randomly. The initial parameters were the same as those presented above.

Statistical analysis

All values were reported as mean \pm s.d. For all variables the hypothesis of a normal distribution was tested with the Shapiro Wilk W-test for small samples (performances) and the Kolmogorov test for the large samples (training loads) (Wetherill *et al.*, 1986; Sen and Shrivastava, 1990). The variations in performance and in the content between each training phase were evaluated with a multiple paired *t*-test with Bonferonni's correction. The training content was compared between each training group using a nonparametric Mann-Whitney U test. Bartlett test was used to verify homoscedasticity of performances. All statistical analyses were completed using Statistica 5.1. (Statsoft, Tulsa, OK, USA). Statistical significance was accepted at $P < 0.05$.

Results

Training characteristics and performances

For the whole group of swimmers, training volume measured during a season was 1675 ± 215 km (mean \pm s.d.). During the entire study period, 13 ± 2 performances were recorded for each swimmer (see Table 1). Coefficient of variation (CV) of the actual performances was $2.0 \pm 0.4\%$. The SS group performed more I1 training and less I2 training than the IS group ($P < 0.05$). The MDS group performed more I4 training than the IS group ($P < 0.05$) (see Table 2). Training volume equal to I2, decreased between the volume and the intensity phase ($P < 0.05$), whereas training volume equal to I4 and strength training (I7) increased. Total meters swum, training volumes equals to I1 and I2 and strength training (I6, I7) decreased between the intensity and the taper phase ($P < 0.05$), whereas training volumes equals to I3, I4 and the maximal intensity (I5) remained constant (see Table 3).

*****Table 3 near here*****.

Performances improved $2.24 \pm 1.24\%$ ($P < 0.05$) during the taper phase. The pattern of training loads and performances during a season exhibited an undulating trend (Fig. 1). The predicted peak performance was in weeks 61 and 62, that is to say three or four weeks after the end of the taper period and after complete cessation of training.

**** Figure 1 near here ****

Performances estimation during the rest period

For the whole group, the time to peak performance was 17 ± 9 days (range 7-35 days) after the end of the training period.

Goodness of fit and accuracy

The relationship between training and performance (using the original estimation method) was significant in all subjects: $R^2 = 0.79 \pm 0.13$, $P < 0.05$. ASE for all subjects was $1.05 \pm 0.63\%$. 95% CI of the parameters and mean 95% CI width for the modelled performances are presented in Table 4. 95% CI of the parameters t_n , and t_g are presented in table 5. The mean CV were 32 ± 14 ; 42 ± 16 ; 64 ± 22 ; 98 ± 32 ; 44 ± 22 ; $41 \pm 22\%$ for τ_a , τ_f , k_a , k_f , t_n , t_g respectively.

**** Table 4 near here ****

****Table 5 near here ****

Ill-conditioning

Correlation between parameter estimates (mean absolute values \pm s.d.) are shown in Table 6. Parameters τ_a and τ_f were highly correlated (0.99 ± 0.01). A high correlation was also found between k_a and k_f (0.91 ± 0.13). Finally, the fatigue magnitude factor was correlated with the fitness and fatigue decay time constants (0.75 ± 0.30 and 0.76 ± 0.27 , respectively).

**** Table 6 near here ****

Stability

The values of the Banister model parameters computed for each athlete with all performances minus one are shown in Table 7.

**** Table 7 near here ****

No significant differences were found between the results of Table 3 (all performances) and Table 7 (all performances minus one).

Discussion

The main findings of the present study were:

1. For the whole group, the time to peak performance was 17 days (in average), after the end of the training period and the main competition of the season.
2. Banister model parameters exhibited wide variability. For illustration, the 95% CI for t_n and t_g were, in average, (7, 35) and (25, 61) days, respectively. In addition, the coefficients of variation for the parameter estimates calculated from 1000 bootstrap replicates were greater than 30%. Conversely, the variability in modelled performances was reasonably small.
3. The Banister model appears to be stable since no significant differences in the parameters were found when the computation was performed with one performance less.

In the present study, the characteristics of training and performances were similar to those reported previously (Mujika *et al.*, 1995, 1996a; Stewart and Hopkins, 2000; Avalos *et al.*, 2003). The mean positive and negative decay time constant ($\tau_a = 38 \pm 16$ days; $\tau_f = 19 \pm 11$ days) were close to those reported in endurance athletes (Mujika *et al.*, 1996a; Morton, 1997; Bannister *et al.*, 1999). It is noteworthy that the parameter values and the determination coefficients are near those reported by Mujika *et al.* (1995, 1996a,b) in a study of similar athletes (international-level

swimmers) over the same duration (one season). However, these values were also close to those reported in studies with different types of athletes. For example, the present decay constant time for the fitness function is similar to the values (~40 days) calculated with less-fit athletes (Morton *et al.*, 1990; Busso *et al.*, 1991). It is therefore unlikely that the range of the parameter values would be similar, irrespective of the quality of the athletes. The mean standard error values for all subjects were less than the difference in performance between the first and the last swimmer in the Athens Olympic Games finals. But they were less accurate than those reported by Avalos *et al.* (2003) who used a linear mixed model (1.05 ± 0.63 % vs. 0.33 ± 0.11 %).

Training and performance quantification

Other methods were applied to quantify the training load and performance. The total training load was also expressed as the summation of the volume in each training level weighed by a coefficient according to relationship between the swimming velocity and the lactate level (Mujika *et al.*, 1996a,b). Performance was also modelled using a logarithm transformation, expressed as a percentage of the world record as well as the criterion points scale proposed by Morton *et al.* (1990). These different methods produced a less reliable adjustment and a significant change in the k_a parameter, expressing the sensitivity of the model to the scale used.

Performance prediction during the rest period

For the whole group, the modelled performance peaks occurred approximately 17 ± 9 days (range 7-35 days), after training ceased. Indeed, these results are consistent with those of Morton in 1991 who, using the original dose-response model of Calvert *et al.* (1976), studied by simulation the effects of variation in ten parameters (four from the dose-response model and six describing the differing shape of

several seasonal training profiles). These authors reported that peak performance occurs on average 23 ± 34 days after the end of training. As the season ended with the most important competition of the year, these results are undesirable from a practical point of view: it would have been more logical for performance to peak during the key competition, and not some 17 days later. These results cannot likely be attributed to poor tapering. The taper phase lasting three weeks was characterised by an approximate 50% decline in training volume and by a decrease in low intensity and dry land training as well as maintained high-intensity as recommended in the literature (Mujika and Padilla, 2003). In addition, performances improved $2.24 \pm 1.24\%$ during the taper phase which is equivalent to the $2.2 \pm 1.5\%$ reported by Mujika *et al.* (2002) during the final 3 weeks of training leading to the Sidney Olympic Games and greater than the 1.48% and 2.07% reported by Bonifazi *et al.* (2000) in male swimmers during two seasons. Moreover, the results of the present study are not in agreement with work carried out on short-term detraining (stopping training for 2 to 4 weeks), which generates a rapid decline in maximal oxygen uptake (VO_{2max}) (Mujika and Padilla, 2000) and decreased swim power, *i.e.* the ability to apply force during swimming (Neufer *et al.*, 1987). It is noteworthy however that these qualities have been shown to be strongly linked with the level of swimming performance (Hawley *et al.*, 1992 ; Wakayoshi *et al.*, 1995). The fact that the changes in the parameters of the present model were not taken into account (time-unvarying model) may also explain why the modelled performance peaks occurred such a long time after training had ceased. With a time-varying model, Busso *et al.* (2002) reported a decrease in t_n and t_g when training frequency was reduced. The time needed to recover performance levels after a training session increased from 0.9 ± 2.1 days at the end of low-frequency

training to 3.6 ± 2.0 days at the end of high-frequency training. The same author used a non-linear model introducing a variable to account for training-related changes in the magnitude and duration of exercise-induced fatigue, reported a decrease in time to peak performance when the training load was reduced from 37 days for a daily training load of 500 a.u. to 7 days for 300 a.u. (Busso, 2003). However, in the latter two models, estimates must be provided for 6 parameters implying the need for at least 15 performances per parameter (Stevens, 1986), *i.e.* 90 performances – totally unworkable under real sporting conditions (Mujika *et al.*, 1996a,b; Millet *et al.*, 2002; Avalos *et al.*, 2003).

Another problem concerns the method used to calculate the global training load, based on a summation of the different types of training. Training in elite swimmers comprises a wide range of diversified exercises (aerobic and anaerobic training, speed exercises, strength training). These different types of training have to be individualised depending on age, training background, individual profiles and specialty (Mujika *et al.*, 1996a; Stewart and Hopkins, 2000; Avalos *et al.*, 2003). For example, in our study sprint swimmers performed a larger proportion of speed training compared with middle distance swimmers who had a higher proportion of $\approx 4 \text{ mmol}\cdot\text{l}^{-1}$ training. The nature of the immediate and long-term training effects of these different exercises on the organism are so diverse that grouping them together or considering them as making up one single training stimulus would be unrealistic (Banister *et al.*, 1975). A similar overall training load may correspond to two very different types of training. (There is a compensation in training volume in each intensity level). Accordingly, Taha and Thomas (2003) argued that the Banister model implicitly assumed that the performance activity matches the training activity and therefore does not consider the specificity of training. Furthermore,

classification in 5 intensity levels may be insufficient when compared to the large number of different training variables used by the coaches in their programmes such as arm and leg training, training in the four styles, and technical training (Mujika *et al.*, 1996b; Stewart and Hopkins, 2000; Avalos *et al.*, 2003). In addition, training for start and turn, which account for about 30% of the event time (Thomson and Haljand, 2000) was not taken into consideration for calculating load for the Banister model. In fact several authors pointed out the multi-faceted nature of performance excellence (Banister *et al.*, 1975; Morton *et al.*, 1990; Mujika *et al.*, 1996a ; Avalos *et al.* , 2003). Psychological, nutritional (Banister *et al.*, 1975; Morton *et al.*, 1990) and technical factors (Toussaint and Hollander, 1994; Wakayoshi *et al.*, 1995) also affect performance. These two authors demonstrated that swimming efficiency, a factor that cannot be quantified in overall load, is a major factor in swimming performance. Nevertheless, the impacts of the various types of training loads on performance have an upper limit above which training does not elicit further adaptation of the subjects (Morton *et al.*, 1997). Hence, the long-term and cumulative effects of training, that may be evidenced some macro-cycles after, may not have been taken into account (Counsilman and Counsilman, 1991; Werchoschanski, 1992; Avalos *et al.*, 2003).

Variability in the parameters of the Banister model

The present study showed that the 95% CI in all parameters was large, in particular, for t_n and t_g , 19 (7, 35) and 43 (25, 61) days, respectively. The 95% CI for the modelled performances was reasonably small ($1.89 \pm 0.42\%$), representing from a practical point of view ~ 1 s for a 100-m event performed in 55 s. CI provides an interval of reasonable estimates, where the width of the interval is determined by the uncertainty in the point estimate. The greater the uncertainty, the wider the CI

(Efron and Tibshirani, 1993; Huet *et al.*, 1996). In addition, the coefficients of variation for the parameter estimates were high (greater than 30%) (Arsac *et al.*, 2004). Parameter variability was much larger compared to the variability in time-dependent parameters reported by Busso *et al.* (1997) and Busso (2003), suggesting, unlike the latter study, that some methodological issues could be involved.

Study of the correlation matrix of parameter estimates revealed ill-conditioning problems, which are known to affect the accuracy of parameter estimates. Ill-conditioning and its effects are well known in linear regression (collinearity) (Wetherill *et al.*, 1986; Sen and Shrivastava, 1990). In non-linear regression, the problem is more complex, and different types of ill-conditioning can be identified (Bates and Watts, 1988; Belsley, 1991). Thus, in the linear case, the inaccuracies of parameter estimates and estimated responses are generally comparable, whereas in the non-linear case, inaccuracy of parameter estimates (estimator conditioning) and inaccuracy of response estimation (data conditioning) can differ (Belsley, 1991). In the present study the 95% CI for parameters was very large, whereas the 95% CI for the modelled performances was reasonably small.

The origins of the present ill-conditioning can be explained by:

- 1) A poor sample size. The most direct and obvious means for improving conditioning is through the collection and use of additional data. Unfortunately, the remedy of obtaining new data is rarely possible in studies dealing with elite athletes, having only a few precious observations. Furthermore, even if new data were obtainable, there is often no guarantee that they will be consistent with the

original data or that they will indeed provide independent information (Belsley, 1991).

2) Interaction between parameters. If parameters are inter-dependent, their interpretation is conditioned and their practical use becomes useless. Several sets of parameters can be the (best) solution for a given loads and performances data set.

3) Misspecification of the model (Bates and Watts, 1988; Huet *et al.*, 1996). Some hypotheses are supposed in the regression problem: normality, homoscedasticity and independence of errors (regarding the random structure), and the Banister model function (regarding the deterministic structure). If one of these hypotheses is false, the model will be misspecified. As a consequence of ill-conditioning, the estimation of the parameters t_n and t_g was very inaccurate. This is supported by the findings of Fitz-Clark *et al.* (1991) who reported that small (10-15%) changes in τ_f , k_a and k_f induced large variations in t_n and t_g , (41% and 21%, respectively). Thus for elite swimmers, the use of these parameters to provide valuable information for the understanding of individual responses to training and to develop individual training schedules from observational data appears to be hazardous.

Possible solutions to variability

Penalization techniques, based on decreasing variability to improve accuracy, are extensively used to resolve ill-conditioning problems in linear regression (Wetherill *et al.*, 1988; Sen and Shrivastava, 1990). Subset selection and ridge regression are the two main penalization procedures. The former consists in selecting a subset of the most relevant variables (Sen and Shrivastava, 1990; Breiman, 1996). Its adaptation to the present problem consists in selecting a subset of parameters. The studies of Busso *et al.* (1991) and Busso (2003) can be understood from a parameter subset selection viewpoint. Thus, the goodness of fit of models with

different numbers of components (*i.e.* different numbers of parameters) was examined.

Ridge regression imposes a constraint on parameters (Wetherill *et al.*, 1988; Sen and Shrivastava, 1990; Breiman, 1996). To date, ridge regression has not been applied to sport data and only few studies have dealt with the adaptation of ridge regression to non-linear regression (see, for example, Minor *et al.*, 1996; O'Sullivan and Saha, 1999; Zhou *et al.* 2002).

Short of new data, the introduction of appropriate prior information is another available solution to the ill-conditioning problem. Procedures to introduce prior information are, for example, mixed-estimation techniques (Belsley, 1991; Davidian and Giltinan, 1995). An application of mixed models to sport data was proposed by Avalos *et al.* (2003).

The re-parameterization of the Banister model may improve conditioning (Bates and Watts, 1988; Huet *et al.*, 1996). Since a high correlation between the fitness and fatigue decay time constants and between the fitness and fatigue magnitude factors was found, dissymmetrizing fitness and fatigue functions may overcome the parameters correlation. For example, one of the decay time constants and one of the magnitude factors could be inversed: $v_a = 1/\tau_a$, $l_a = 1/k_a$. These operations do not affect the interpretation of parameters (as initial parameters can be easily computed from the new ones), however, they may be numerically advantageous.

Misspecification problems are probably involved in the present study: (i) the Banister model does not take into account the possible dependence between performances, a typical problem in longitudinal data studies (Avalos *et al.*, 2003); (ii) In the Banister model, training impulses are proportional to the training loads then, greater loads induce more fitness and fatigue acquisition. But previous studies

reported that the impact of training loads on performance may have an upper limit above which training does not elicit further adaptation of the organism (Fry *et al.*, 1991; Morton, 1997); (iii) The procedure assumes the parameters remain constant over time, an assumption that is not consistent with observed time-dependent alterations in responses to training (Busso *et al.*, 1997; Avalos *et al.*, 2003; Busso, 2003).

Stability

The Banister model estimated by the non-linear least squares method seemed to be stable since no significant differences were found when the computation was performed with one performance less. The change in the temporal parameters were ~1-3 days for most of the subjects. However, in one subject (n° 6) the difference in τ_f was 8 days. Moreover, the values of t_n and t_g are greatly modified by little change in τ_a or τ_f . For example, in subject 1, the values of t_n and t_g were decreased from 12 to 7 days and 24 to 15 days, respectively, when τ_f decreased by three days and in subject 9 the values of t_n and t_g were increased from 7 to 17 days and 54 to 66 days, when τ_a decreased by 3 days and τ_f by 2 days. These changes may have important practical implications in planning training loads.

Therefore proposing a single set of parameters might be dubious. Breiman (1996) showed that the averaged estimator of the bootstrap estimations is more stable than the original one. Stabilizing non-linear methods can give non-linear estimators with improved accuracy. This technique was used in the present study but a large variability was also observed in all parameters of the model and accuracy was not improved.

Perspectives for future study and use of the Banister model.

Further studies should be conducted to determine whether the parameters estimation of the Banister model would be more accurate under standardised experimental conditions. Such standardised conditions would allow multiple recordings of performance and use of a single type of exercise, e.g. as in the study reported by Busso (2003) where exercises were limited to ergometer cycling. The accuracy and the stability of the performance estimations observed in the present study suggest that the pattern of performance changes could be assessed as a function of training load. Nevertheless, the mean error of performance estimates was greater than reported by Avalos *et al.* (2003) suggesting that the results need to be validated experimentally by another complementary method

Other procedures

Another possibility is to use nonparametric regression. Edelman-Nusser *et al.* (2002) suggested applying a non-parametric model (multilayer perceptron neural networks) to model training load-performance relationships. These authors used an unconventional method giving a surprisingly small prediction error (0.04%). Indeed, the model was fitted with data pertaining to one particular swimmer and then used to predict the performance of a different swimmer, despite the consensus that reigns about the singularity of training responses (Mujika *et al.*, 1996a,b; Busso *et al.*, 1997; Millet *et al.*, 2002; Avalos *et al.*, 2003; Busso, 2003).

Neural networks may be particularly useful when the primary goal is outcome prediction, but these techniques are a “black box” and have limited ability to explicitly identify possible causal relationships. Thus, the interpretation of the results obtained is not straightforward. However, in any given parametric problem, the parameters have meaningful interpretations (Hastie *et al.*, 2001).

Recently, Perl (2002) developed a dynamic meta model based on two antagonistic systems (two internal buffer potentials: one positive and one negative, which influence the performance potential alternately). This meta model seems conceptually very rich because it takes into account the collapse effect in the wake of an overloaded training period, atrophy following a period of detraining, not to mention the long-term behaviour of the training-performance relationship (Perl *et al.*, 2001). However, to date, no statistical study has validated the quality of this meta model.

Finally, few authors have used multiple regressions to create a model for the relationship between training and performance. Mujika *et al.* (1996b), using stepwise regression, reported a very close match with the Banister model. Multiple regressions make it possible to integrate different training loads as independent variables and can take into account the effects of load sequences over time with short-term deferred effects during the three weeks that precede the performance (weeks 0, -1, -2), intermediate effects (weeks -3, -4 and -5) and long-term effects (weeks -6, -7, -8), as indicated by Avalos *et al.* (2003). The results we obtained with this method, using data from the present study, showed improved statistical accuracy in estimated parameters and modelled performances (more accurate CI). Moreover, in multiple regression each training variable could be transformed for instance by a quadratic function (or higher order functions) to take into account a potential parabolic relationship between the quantity of training loads and performance (Sen and Shrivastava, 1990). It is also possible to take into account any effects of interaction by associating the different input variables (Sen and Shrivastava, 1990).

Conclusion

The aim of this study was to assess whether the Banister model could be used to monitor the training process in the “real world”, i.e. with real international-level performances. The present study assessed the goodness of fit, accuracy and stability of the Banister model as applied to training loads and performances in elite swimmers. The model showed substantial variability in its parameters, making it imprecise. To conclude, it seems inappropriate to use these parameters to monitor the training process in elite swimmers. Nevertheless, the variability in modelled performances was reasonably small and the Banister model appeared to be stable. Further research should be conducted to determine whether associating these Banister model qualities with other methods of modelling could provide pertinent information for monitoring training.

References

- American College of Sports Medicine. (2002). Progression models in resistance training for healthy adults. *Medicine and Science in Sports and Exercise*, **34**, 364-380.
- Arsac, L. M., Thiaudière, E., Diolez, P. and Gerville-Réache, L. (2004). Parameter estimation in modeling phosphocreatine recovery in human skeletal muscle. *European Journal of Applied Physiology*, **91**, 419-424.
- Atkinson, G. and Nevill, A.M. (1998). Statistical methods for assessing measurement error (reliability) in variables relevant to sports medicine. *Sports Medicine*, **26**, 217-238.
- Avalos, M., Hellard, P. and Chatard, J.C. (2003). Modeling the training-performance relationship using a mixed model in elite swimmers. *Medicine and Science in Sports and Exercise*, **35**, 838-846.
- Banister, E.W., Calvert, T.W., Savage, M.V. and Bach, A. (1975). A system model of training for athletic performance. *Australian Journal of Sports Medicine*, **7**, 170-176.
- Banister, E.W., Carter, J.B. and Zarkadas, P.C. (1999). Training theory and taper: validation in triathlon athletes. *European Journal of Applied Physiology*, **79**, 182-191.
- Bartley, R. (1997). The use and abuse of statistics in sport and exercise sciences. *Journal of Sports Sciences*, **15**, 1-2.
- Bates, D. M. and Watts D. G. (1988). *Nonlinear regression analysis and its applications*. New York: John Wiley & Sons, Inc., pp. 52-53; 67-70; 80-81; 90-92.
- Belsley, D. A. (1991). *Conditioning diagnostics. Collinearity and weak data in regression*. New York: John Wiley & Sons, Inc., 1991, pp. 342-378.

- Bonifazi, M., Saldella, F. and Luppo, C. (2000). Preparatory versus main competitions : differences in performances, lactate responses and pre-competition plasma cortisol concentrations in elite male swimmers. *European Journal of Applied Physiology* **82**: 368-373.
- Breiman, L. (1996) Heuristics of instability and stabilization in model selection. *Annals of Statistics*, **24**, 2350-2383.
- Busso, T. (2003). Variable dose-response relationship between exercise training and performance. *Medicine and Science in Sports and Exercise*, **35**, 1188-1195.
- Busso, T., Hakkinen, K., Pakarinen, A., Carasso, C., Lacour, J. R., Komi, P. V. and Kauhanen, H. (1990). A systems model of training responses and its relationship to hormonal responses in elite weight-lifters. *European Journal of Applied Physiology*, **61**, 48-54.
- Busso, T., Carasso, C. and Lacour, J.R. (1991). Adequacy of a systems structure in the modeling of training effects on performance. *Journal of Applied Physiology*, **71**, 2044-2049.
- Busso, T., Candau, R. and Lacour, J.R. (1994). Fatigue and fitness modelled from the effects of training on performance. *European Journal of Applied Physiology*, **69**, 50-54.
- Busso, T., Denis, C., Bonnefoy, R., Geyssant, A. and Lacour, J.R. (1997). Modeling of adaptations to physical-training by using a recursive least-squares algorithm. *Journal of Applied Physiology*, **82**, 1685-1693.
- Busso, T., Benoit, H., Bonnefoy, R., Feasson, L. and Lacour, J.R. (2002). Effects of training frequency on the dynamics of performance response to a single training bout. *Journal of Applied Physiology*, **92**, 572-580.

Calvert, T.W., Banister, E.W. and Savage, M.V. (1976). A systems model of the effects of training on physical performance. *SMC Systems, Man and Cybernetics*, **2**, 94-102.

Counsilman, B.E. and Counsilman, J.E. (1991). The residual effects of training. *Journal of Swimming Research*, **7**, 5-12.

Davidian, M. and Giltinan D.M. (1995). *Nonlinear models for repeated measurement data*. London: Chapman and Hall, pp. 26-43; 63-71.

Edelmann-Nusser, J., Hohmann, A. and Henneberg, B. (2002). Modeling and prediction of competitive performance in swimming upon neural networks. *European Journal of Sport Science*, **2**, 1-12.

Efron, B. and Tibshirani, R. (1993) *An introduction to the Bootstrap. Monographs on statistics and applied probability*. New York: Chapman and Hall, pp. 237-238; 242-243.

Fitz-Clarke, J.R., Morton, R.H. and Banister, E.W. (1991). Optimizing athletic performance by influence curves. *Journal of Applied Physiology*, **71**, 1151-1158.

Fry, R.W., Morton, A.R. and D. Keast. (1992). Periodisation of training stress-A review. *Canadian Journal of Sport Sciences*, **17**, 234-240.

Hastie, T., Tibshirani, R. and Friedman, J. (2001). *The elements of statistical learning. Data mining, inference and prediction. Springer Series in Statistics*. New York: Springer.

Hawley

Huet, S., Bouvier, A., Gruet, M. A. and Jolivet, E. (1996). *Statistical tools for nonlinear regression: a practical guide with S-PLUS examples*. New York, PA: Springer-Verlag, pp.12-13; 30-38; 89-90; 111-115.

- Millet, G.P., Candau, R.B., Barbier, B., Busso, T., Rouillon, J.D. and Chatard, J.C. (2002). Modelling the transfers of training effects on performance in elite triathletes. *International Journal of Sports Medicine*, **23**, 55-63.
- Minor, J.M., Namini, H. and Watson, G.A. (1996). Generalized ridge analysis with application to population pharmacokinetics/dynamics. *Journal of Biopharmaceutical Statistics*, **6**, 105-114.
- Morton, R.H. (1997). Modelling training and overtraining. *Journal of Sports Sciences*, **15**, 335-340.
- Morton, R.H. (1991). The quantitative periodization of athletic training: a model study. *Sports Medicine Training and Rehabilitation*, **3**, 19-28.
- Morton, R.H., Fitz-Clarke, J.R. and Banister, E.W. (1990). Modelling human performance in running. *Journal of Applied Physiology*, **90**, 1171-1177.
- Mujika, I. and Padilla, S. (2003). Scientific bases for precompetition tapering strategies. *Medicine and Sciences in Sports and Exercise* **35**, 1182-1187.
- Mujika, I., Padilla, S. and Pyne, D. (2002). Swimming performance changes during the final 3 weeks of training leading to the Sidney 2000 Olympic games. *International Journal of Sports Medicine*, **23**, 582-587.
- Mujika, I. and Padilla, S. (2000). Detraining: loss of training-induced physiological and performance adaptations. Part I: short term insufficient training stimulus. *Sports Medicine*, **30**, 79-87.
- Mujika, I., Busso, T., Lacoste, L., Barale, F., Geyssant, A. and Chatard, J. C. (1996a). Modelled responses to training and taper in competitive swimmers. *Medicine and Science in Sports and Exercise*, **28**, 251-258.

- Mujika, I., Chatard, J.C., Busso, T. and Geyssant, A. (1996b). Use of swim-training profiles and performance data to enhance training effectiveness. *Journal of Swimming Research*, **11**, 23-29.
- Mujika, I., Chatard, J. C., Busso, T., Geyssant, A., Barale, F. and Lacoste, L. (1995). Effects of training on performance in competitive swimming. *Canadian Journal of Applied Physiology*, **20**, 395-406.
- Neufer, P.D., Costill, D.L., Fielding, R.A., Flynn, M.G and Kirwan, J.P. (1987). Effect of reduced training on muscular strength and endurance in competitive swimmers. *Medicine and Science in Sports and Exercise*, **19**, 486-490.
- O'Sullivan, F. and Saha, A. (1999). Use of ridge regression for improved estimation of kinetic constants from PET data. *IEEE Transactions on Medical Imaging*, **18**, 115-125.
- Perl, J., Dausher, P. and Hawlitzky, M. (2001). On the long term behaviour of the performance potential-metamodel Perpot. *International Journal of Computer Science in Sport*, **1**, 56-57.
- Perl, J. (2002). *Adaptation, antagonism and system dynamics*. In The Multidisciplinary Series of Physical Education and Sport Science (Edited by G. Ghent, D. Kluka & D. Jones), **4**, pp. 105-125. Meyer & Meyer Sport.
- Sen, A., and Shrivastava, M. (1990). *Regression analysis: theory, methods, and applications*. New York, Berlin, Paris. Springer Verlag, pp. 37-40; 60-62; 100-111; 218-220; 233-238; 256-262; 298-310.
- Stevens, J. (1986). *Applied multivariate statistics for the social sciences*. Hillsdale, NJ: Erlbaum, pp.72.
- Stewart, A.M. and Hopkins, W.G. (2000). Seasonal training and performance of competitive swimmers. *Journal of Sports Sciences*, **18**, 873-884.

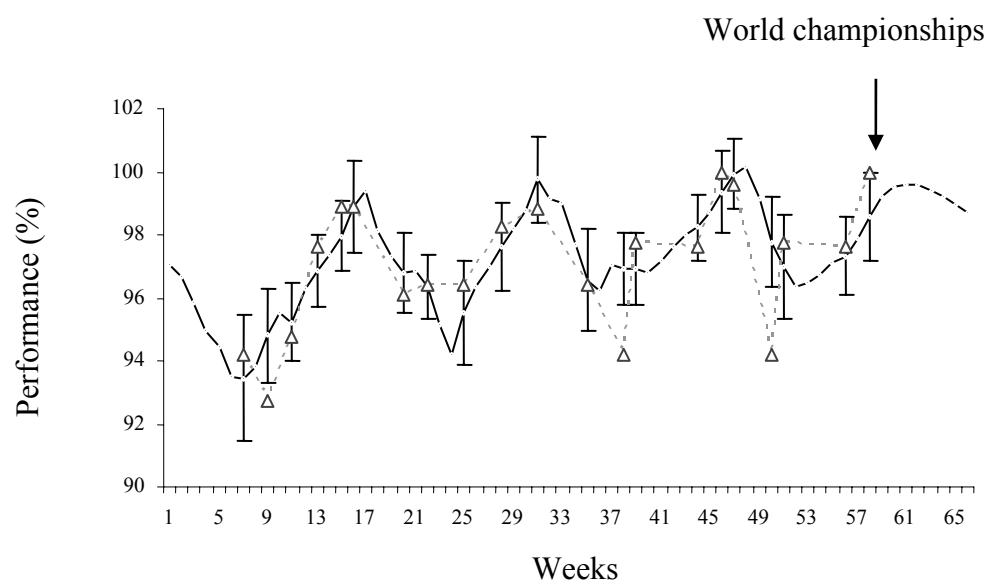
- Taha, T. and Thomas, S.G. (2003). Systems modeling of the relationship between training and performance. *Sports Medicine*, **33**, 1061-1073.
- Thomson, K.G., Haljand, R. and Mc Laren, D.P. (2000). An analysis of selected kinematic variables in national and elite male and female 100-m and 200-m breaststroke swimmers. *Journal of Sports Sciences*, **18**, 421-431.
- Toussaint, H. M. and Hollander, A. P. (1994). Energetics of competitive swimming - Implications for training-programs. *Sports Medicine*, **18**, 384-405.
- Wakayoshi, K., D'Acquisto, L.J., Cappaert, J.M. and Troup, J.P. (1995). Relationship between oxygen uptake, stroke rate and swimming velocity in competitive swimming. *International Journal of Sports Medicine*, **16**, 19-23.
- Werchoschanski, J. W. (1992). *Efficient training. (L'entraînement efficace)*. Paris, PA: Presses Universitaires de France, pp. 38, 43-44, 124-125, 166-167.
- Wetherill, G.B., Duncombe, P., and Kenward, M. *Regression analysis with applications*. London, New York: 1986, pp. 82-86; 102-104; 180-182; 226-227; 240-246.
- Wolfarth, B., Rivera, M.A., Oppert, J.M., Boulay, M.R., Dionne, F.T., Chagnon, M., Gagnon, J., Chagnon, Y., Perusse, L., Keul, J., and Bouchard, C. (2000). A polymorphism in the alpha 2a-adrenoceptor gene and endurance athlete status. *Medicine and Sciences in Sports and Exercise*, **32**, 1709-1712.
- Zhou, Y., Huang, S. C., Bergsneider, M. and Wong, D. F. (2002). Improved parametric image generation using spatial-temporal analysis of dynamic PET studies. *Neuroimage*, **15**, 67-707.

LEGEND OF FIGURES

Figure 1 – Example for subject 3 (Olympic finalist): **(a)** Modelled (line) and actual performances (dotted line with triangles). 95% CI for modelled performances are also presented. Performances on vertical axis were expressed in percentage of the personal record $\min_t(P_t)$ and computed as $p_t = \frac{\min_t(P_t)}{P_t} * 100$. **(b)** Training loads on vertical axis are expressed as a percentage of the maximal training load performed by the subject during the course of the study. Time in horizontal axis is expressed in weeks.

Figure 1

(a)



(b)

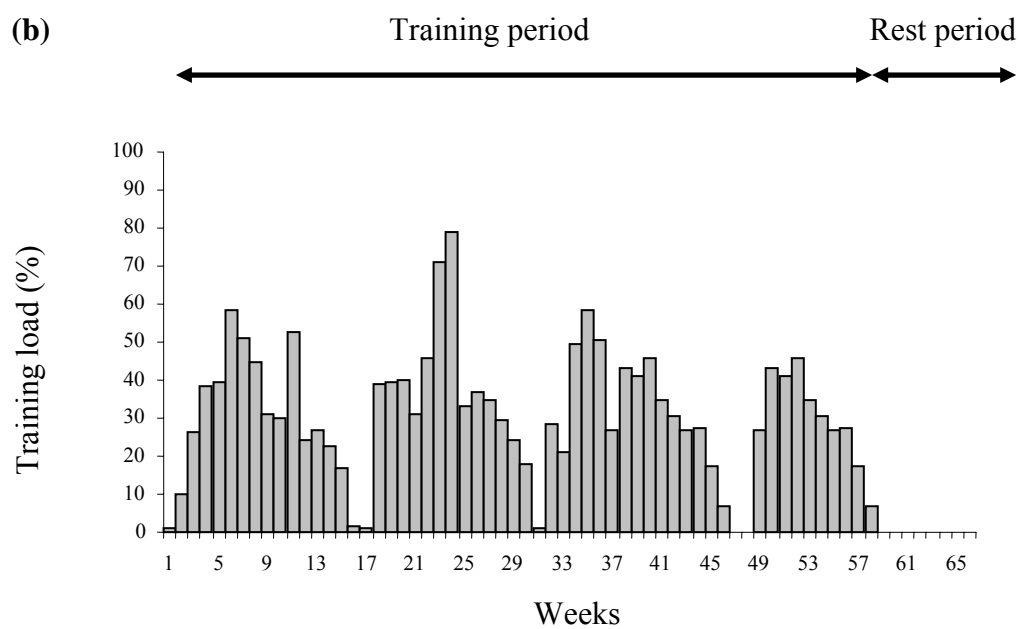


Table 1. Selected characteristics of the subjects training over the one-year period studied.

S	G	Age (years)	Height (cm)	Weight (kg)	Training (km)	Event	Best Perf	Number of Perf's	CV of Perf (%)
1	F	24	168	61	1402	100 Free	00:55:65	11	2.3
2	F	21	173	62	1856	200 Butt	02:10:8	12	1.6
3	F	26	179	59	1677	200 Free	01:59:86	21	2.2
4	M	27	185	84	1751	200 Medl	02:01:83	12	1.9
5	M	23	181	81	1340	100 Breast	01:03:51	14	1.8
6	F	26	168	50	1477	200 Back	02:15:00	18	2.4
7	M	20	186	80	1815	100 Free	00:51:5	12	2.6
8	F	19	167	52	1916	200 Free	02:03:51	11	1.6
9	M	23	188	84	1843	400 Free	03:53:42	13	1.5
Mean		23.2	177.2	68.1	1675			13.2	2.0
s.d.		2.8	8.4	14.0	215			2.4	0.4

S = Subjects, G = Gender, Perf = Performance, CV = coefficient of variation, Free = Freestyle, Butt = Butterfly, Medl = Medley, Breast = Breaststroke, Back =Backstroke. Training (km) = Total km swum during the period concerned.

Table 2. Annual swimming volume percentage and annual dry land training percentage (including rest periods) for each intensity level and group.

Intensity Level	SS		IS		MDS	
	IS	MDS	SS	MDS	SS	IS
I1	*	-	*	-	-	-
(%· yr ⁻¹)	69.3 (5.4)		57.3 (4.9)		48.1 (4.1)	
I2	*	-	*	-	-	-
(%· yr ⁻¹)	21.6 (2.2)		34.6 (4.5)		44.1 (5.4)	
I3	-	-	-	-	-	-
(%· yr ⁻¹)	4.6 (2.2)		4.7 (0.4)		3.8 (1.1)	
I4	-	-	-	#	-	#
(%· yr ⁻¹)	2.3 (0.2)		2.5 (0.2)		3.3 (0.3)	
I5	*	-	*	-	-	-
(%· yr ⁻¹)	2.3 (0.2)		0.9 (0.1)		0.7 (0.2)	
I6	-	-	-	-	-	-
(%· yr ⁻¹)	41.7 (20.2)		37.8 (26.7)		29.6 (0.7)	
I7	-	-	-	-	-	-
(%· yr ⁻¹)	58.5 (20.2)		62.2 (27.7)		70.4 (0.7)	

Values are mean (s.d.). SS = sprint swimmers, IS = intermediate swimmers, MDS = middle distance swimmers. * Significant difference between SS and IS, for each intensity level ($P \leq 0.05$). # Significant difference between IS and MDS, for each intensity level ($P \leq 0.05$). -Not significantly different. Mann Whitney U test for nonparametric distribution was used.

Table 3. Respective contents of the volume, intensity and taper phases during the fourth training cycle (Before the main competition of the year).

	Volume phase		Intensity phase		Taper phase	
Total (m)	55200 ± 8772		49500 ± 7340	**	28900 ± 3780	++
I1 (m)	19108 ± 3375		24637 ± 2920		18841 ± 2941	+
I2 (m)	33800 ± 6229	£	21550 ± 5340	**	7000 ± 973	++
I3 (m)	1333 ± 472		1125 ± 382		1366 ± 502	
I4 (m)	533 ± 227	£	1650 ± 443	*	1291 ± 483	
I5 (m)	425 ± 197		537 ± 176		478 ± 181	
I6 (mn)	12 ± 4		8 ± 3	**	3 ± 1	++
I7 (mn)	15 ± 5	£	28 ± 7	*	12 ± 4	++

Intensity levels for swim workouts: below (I1 $\approx 2 \text{ mmol}\cdot\text{l}^{-1}$), equal to (I2 $\approx 4 \text{ mmol}\cdot\text{l}^{-1}$), slightly above (I3 $\approx 6 \text{ mmol}\cdot\text{l}^{-1}$) the onset of blood lactate accumulation, respectively; I4 $\approx 10 \text{ mmol}\cdot\text{l}^{-1}$; I5 = maximal intensity; I6 and I7 consisted in 40-70% and 70%-100% of 1 maximal repetition strength training. Results are expressed in meters and minutes. £ Significant differences between volume and intensity phases ($P < 0.05$). +, ++ Significant differences between volume and taper phases ($P < 0.05$ and $P < 0.01$, respectively). *, ** Significant differences between intensity and taper phases ($P < 0.05$ and $P < 0.01$, respectively).

Table 4. Banister model parameters measured over the one-year period studied.

S	p_0	CI p_0	k_a	CI k_a	k_f	CI k_f	τ_a	CI τ_a	τ_f	CI τ_f	R^2	MIW
1	0.92	0.89, 0.95	0.002	-0.040, 0.044	0.016	-0.018, 0.051	40	8, 71	5	-6, 16	0.69*	3.17
2	0.95	0.92, 0.98	0.106	0.022, 0.189	0.129	0.071, 0.185	13	-6, 33	11	7, 15	0.84†	1.99
3	0.97	0.93, 1.01	0.039	0.006, 0.071	0.048	0.016, 0.081	33	8, 57	27	8, 46	0.65*	2.69
4	0.97	0.95, 0.98	0.050	-0.029, 0.068	0.068	0.048, 0.088	27	14, 40	20	15, 25	0.97§	0.80
5	0.98	0.95, 1.01	0.003	-0.040, 0.046	0.022	-0.012, 0.057	41	11, 70	9	-1, 19	0.78†	1.80
6	0.90	0.84, 0.94	0.009	-0.014, 0.034	0.016	-0.009, 0.041	45	18, 71	18	4, 32	0.61§	2.04
7	0.90	0.78, 1.02	0.018	-0.012, 0.046	0.028	-0.003, 0.057	57	39, 75	31	17, 46	0.95§	1.55
8	0.95	0.92, 0.97	0.083	0.019, 0.148	0.112	0.048, 0.176	23	11, 34	16	9, 24	0.92§	1.58
9	0.93	0.90, 0.96	0.010	-0.024, 0.046	0.012	-0.016, 0.042	65	50, 81	38	3, 64	0.73*	2.44
Mean	0.94	0.90, 0.98	0.036	-0.012, 0.077	0.050	0.014, 0.086	38	17, 59	19	6, 32	0.79	1.89
s.d.	0.03		0.038		0.044		16		11		0.13	0.49

S = subjects; p_0 = the initial performance level (in % of the best performance); k_a and k_f = the fitness and fatigue magnitude factors, respectively (in arbitrary units); τ_a and τ_f = the fitness and fatigue decay time constants, respectively (in days). CI = Bootstrap 95% confidence interval of estimated parameters. * Significant, $P < 0.05$; † Significant, $P < 0.01$; § Significant, $P < 0.001$. MIW = Mean Interval Width of 95% confidence interval of modelled performances.

Table 5. Model parameters t_n and t_g computed from k_a and k_f (the fitness and fatigue magnitude factor) and τ_a and τ_f (the fitness and fatigue decay time constant).

S	t_n	CI t_n	t_g	CI t_g
1	7	-2.15	15	-3.34
2	10	1.19	21	9.32
3	31	25.77	61	20.96
4	25	21.28	48	43.53
5	24	6.40	41	16.65
6	12	5.19	35	20.50
7	29	8.51	71	57.85
8	17	12.20	36	28.41
9	15	-14.45	64	34.95
Mean	19	7.35	43	25.61
s.d.	9		19	

S = subjects, t_n = the time to recover performance, t_g = the time to peak performance after training completion. CI = Bootstrap 95% confidence interval of estimated parameters.

Table 6. Correlation between parameter estimates for the Banister model ($n = 9$; mean absolute value \pm s.d.).

Parameters	Correlation
$p_0 - k_a$	0.49 ± 0.27
$p_0 - k_f$	0.31 ± 0.25
$p_0 - \tau_a$	0.41 ± 0.27
$p_0 - \tau_f$	0.41 ± 0.27
$k_a - k_f$	0.91 ± 0.13
$k_a - \tau_a$	0.69 ± 0.26
$k_a - \tau_f$	0.69 ± 0.26
$k_f - \tau_a$	0.75 ± 0.30
$k_f - \tau_f$	0.76 ± 0.27
$\tau_a - \tau_f$	0.99 ± 0.01

p_0 = the initial performance level; k_a and k_f = the fitness and fatigue magnitude factors, respectively; τ_a and τ_f = the fitness and fatigue decay time constants, respectively.

Table 7. Banister model parameters measured over the one-year period studied. One performance chosen randomly was removed in order to assess model stability.

Subjects	p_0	k_a	k_f	τ_a	τ_f
1	0.93	0.002	0.077	39	2
2	0.95	0.023	0.054	17	7
3	0.98	0.032	0.040	33	26
4	0.97	0.035	0.054	28	19
5	0.98	0.003	0.021	41	10
6	0.90	0.053	0.067	47	26
7	0.90	0.020	0.030	55	32
8	0.94	0.079	0.106	23	16
9	0.92	0.011	0.012	62	36
Mean	0.94	0.029	0.051	38	19
s.d.	0.03	0.025	0.029	15	12

p_0 = the initial performance level; k_a and k_f = the fitness and fatigue magnitude factors, respectively; τ_a and τ_f = the fitness and fatigue decay time constants, respectively.